Discussion of “Wealth Inequality and Endogenous Growth” by Byoungchan Lee

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The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System.
Summary of the Paper

- Question: how does rising inequality affect economic growth?
- Model: Aiyagari meets endogenous growth
- Mechanism:
  - Higher wealth inequality ($\sigma_a^2$) decreases consumption intensity ($s_C$)
    - Intuition: rich households save more
  - Lower consumption intensity ($s_C$) decreases productivity growth ($g$)
    - Intuition: smaller market size and less profits from R&D $\rightarrow$ firms reduce R&D effort
- Contribution:
  - Develop an analytically tractable heterogeneous-agent model
  - Incorporate endogenous innovation into an incomplete mkt model
- My assessment: very important topic + very interesting mechanism!
Two Key Equations

▶ Consumption function: (proposition 1)

\[ c = \zeta (x + \eta)^\xi, \quad 0 < \xi < 1 \]

where \( \xi \) determines the concavity.

▶ Consumption intensity

Under specific assumptions, consumption intensity is characterized by consumption function parameters and wealth distribution parameters.

\[ s_C = \frac{\zeta \exp(\xi \mu_a)}{1 - \frac{1}{2} \xi^2 \sigma_a^2} \bigg/ \left[ \frac{\exp(\mu_a)}{1 - \frac{1}{2} \sigma_a^2} - \eta \right] \]

▶ \( \xi \) crucially determines \( \frac{\partial s_C}{\partial \sigma_a^2} \)
Rising inequality reduces consumption intensity → by how much?

- In a basic Aiyagari model, demand intensity does not change much w.r.t. the distribution of wealth.

What drives the difference?

- Aiyagari model understates the saving rate for wealthy HHs.
- Lee (2021) circumvents this problem by estimating ξ from the data.
- Pros and cons?
Model Environment

- **State variables**
  - household asset $b^i_t$
  - transitory component of earnings risk $\epsilon^i_t$
  - permanent component of earnings risk $z^i_t$
  - individual-specific fixed effect $\kappa^i$

- **Income process**
  \[ y^i_t = \epsilon^i_t + z^i_t + \kappa^i \]
  \[ z^i_t = \rho z^i_{t-1} + \eta_t \]

  where $\epsilon_t \sim N(0, \sigma_\epsilon)$ and $\eta_t \sim N(0, \sigma_\eta)$.

- **Financial market**
  \[ b^i_{t+1} \geq -\phi \]

Not in Lee (2021)
Recursive Household Problem

- Household problem

\[ v(b, \kappa, z, \epsilon) = \max_{c, b'} \left\{ \frac{c^{1-\sigma}}{1 - \sigma} + \beta \mathbb{E}_{z', \epsilon'} [v(b', \kappa, z', \epsilon')] \right\} \]

subject to

\[ c + b' = (1 + r)b + wy(\kappa, z, \epsilon) \]
\[ c \geq 0, b' \geq -\phi \]

- Wealth inclusive of income

\[ x = (1 + r)b + wy(\kappa, z, \epsilon) \]
\[ c + b' = x \]
What We Do

- Solve the calibrated Aiyagari model
- Simulate 60,000 households until finding the stationary distribution.
- Question 1: how does demand intensity change w.r.t. inequality?
- Question 2: what is the model implied parameters for the consumption function?
- Question 3: what are the aggregate and distributional consequences if we override the model implied $\xi$ with the estimated $\hat{\xi}$?
Wealth Inequality and Aggregate Consumption

- Quantitative experiment
  - Increase $\sigma_z$ to match the change in the standard deviation of log earnings documented in Song et al. (2019)
  - Fix the total endowment of labor
  - Study the GE effect of the rise in inequality

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>$\sigma_z = 0.14$</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of log earnings</td>
<td>0.84</td>
<td>0.92</td>
<td>0.08</td>
</tr>
<tr>
<td>Top 20% wealth share</td>
<td>0.73</td>
<td>0.82</td>
<td>0.09</td>
</tr>
<tr>
<td>Wealth Gini</td>
<td>0.69</td>
<td>0.78</td>
<td>0.09</td>
</tr>
<tr>
<td>Real interest rate %</td>
<td>4.00</td>
<td>3.87</td>
<td>-0.13</td>
</tr>
<tr>
<td><strong>Aggregate consumption</strong></td>
<td>2.00</td>
<td>2.00</td>
<td><strong>0.26%</strong></td>
</tr>
<tr>
<td><strong>Consumption intensity %</strong></td>
<td>20.21</td>
<td>19.98</td>
<td><strong>-0.23</strong></td>
</tr>
</tbody>
</table>
The Key Step: Consumption Function

In Lee (2021): $\xi$ crucially determines $\frac{\partial s_c}{\partial \sigma_a^2}$
Lee (2021) estimates $\xi$ by a linear regression using PSID
  - $\xi = 0.5$

Aiyagari model understates the saving rate for the rich HHs.
  - Implications: $\xi$ depends on wealth levels

How much is the difference b/t the regression estimated $\xi$ and the model implied $\xi$?
What We Do

▶ Solve the calibrated Aiyagari model
▶ Simulate 60,000 households until finding the stationary distribution.
▶ Question 1: how does demand intensity change w.r.t. inequality?
▶ Question 2: what is the model implied parameters for the consumption function?
▶ Question 3: what are the aggregate and distributional consequences if we override the model implied $\xi$ with the estimated $\hat{\xi}$?
Estimation of $\xi$

- Estimate using all households
- Sort HHs by wealth inclusive of income and estimate for each group
Problem with the Linear Regression

- Using a linear regression, the estimated $\xi$ overstates consumption by poor HHs and understate consumption by wealthy HHs.
- $\xi$ of the rich HH has important implications on consumption intensity.
What We Do

- Solve the calibrated Aiyagari model
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- Question 1: how does demand intensity change w.r.t. inequality?
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Implications on the Aggregate Economy

1. We simulate the economy according to the estimated policy function.
2. In our simulation, from any distribution, without any shocks, wealth concentration increases over time.
3. The distribution might be non-stationary.
4. What are the underlying driving forces?
Final Thoughts

▶ The saving rate for rich HHs in a basic Aiyagari model is lower than the data.
  ▶ In this case, demand intensity does not change much when wealth concentration increases.
▶ Lee (2021) solves this problem by deriving and estimating the consumption function from the data.
  ▶ Pros: more consistent with the data.
  ▶ However, what is the micro-foundation?
▶ Literature: augment Aiyagari model with
  ▶ Non-homothetic preferences: De Nardi (2004) and Straub (2019)
  ▶ Entrepreneurship: Cagetti and De Nardi (2006)
## Appendix: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(A) Assigned parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>( \alpha )</td>
<td>0.36</td>
<td></td>
<td></td>
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<tr>
<td>Risk aversion</td>
<td>( \sigma )</td>
<td>2.00</td>
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<tr>
<td><strong>(B) Calibrated parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount rate</td>
<td>( \beta )</td>
<td>0.94 Real interest rate</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>0.08 Capital to output</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Borrowing limit</td>
<td>( \phi )</td>
<td>0.17 Hhs w/ negative asset %</td>
<td>9.7</td>
<td>9.7</td>
</tr>
<tr>
<td>Variation of fixed effect</td>
<td>( \sigma_\kappa )</td>
<td>0.17 SD of log earnings</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>(C) Estimated parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence of shocks</td>
<td>( \rho )</td>
<td>0.99 Guvenen (2009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of permanent shocks</td>
<td>( \sigma_z )</td>
<td>0.12 Guvenen (2009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of transitory shocks</td>
<td>( \sigma_\epsilon )</td>
<td>0.25 Guvenen (2009)</td>
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<td></td>
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